Square Roots

1^2	=	1
2^2	=	4
3^2	=	9
4^2	=	16
5^2	=	25
6^2	=	36
7^2	=	49
8^2	=	64
9^2	=	81
10^2	=	100
11^2	=	121
12^2	=	144
13^2	=	169
14^2	=	196
15^2	=	225

Basic Concepts

- When a number is multiplied by itself, the resulting product is a ***perfect square***.
- Therefore, that number is the ***square root*** of the perfect square.
- The symbol for square root, **√**, called a ***radical sign***, denotes the ***principal*** or ***nonnegative*** square root. (Although a negative number multiplied by itself also results in a positive perfect square, principal square roots are nonnegative by definition.)
- The expression under the radical sign is called the ***radicand***.
- Therefore: $\sqrt{25} = 5, \sqrt{4} = 2, \sqrt{100} = 10$
- The square root of a negative number is not a real number. No number squared equals a negative product. Example: √-25 has no real number solution.

  However, $-\sqrt{25} = -5$, since the negative sign is outside the radical sign.
- A square root multiplied by itself is equal to the radicand. $\sqrt{4} \bullet \sqrt{4} = 2 \bullet 2 = 4$

Product Property of Square Roots

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

- Therefore , $\sqrt{4 \bullet 25} = \sqrt{100} = 10$ and $\sqrt{4}\sqrt{25} = 2 \bullet 5 = 10$

Simplifying Square Roots

- When the radicand is not a perfect square and does not have a factor that is a perfect square, then it is in ***simplest radical form*** and cannot be computed without a calculator.

- When the radicand is not a perfect square but has a factor that is a perfect square then it can be simplified by finding the square root of the perfect square factor and leaving the remaining factor as the radicand.

  Example: $\sqrt{12} = \sqrt{3 \bullet 4}$ Since 4 is a perfect square, this can be simplified. $\sqrt{4} = 2$, therefore the simplest radical form is $2\sqrt{3}$.

- **Variables with even exponents are always perfect squares**.

  Example: $\sqrt{x^2} = x$ because $x \bullet x = x^2$, $\sqrt{x^6} = x^3$ because $x^3 \bullet x^3 = x^6$

  To solve, find the two equal powers that add up to the exponent in the radicand.

- Variables with odd exponents are not perfect squares, but can be easily simplified.

  Example: $\sqrt{x^3} = \sqrt{x^2} \bullet \sqrt{x} = x\sqrt{x}$, $\sqrt{x^{11}} = \sqrt{x^{10}} \bullet \sqrt{x} = x^5\sqrt{x}$

  If the exponent in the radicand is odd, then subtract 1 from it. Simplify the perfect square factor and leave the remaining variable factor as the radicand.

Square Roots

Quotient Property of Square Roots and Rationalizing the Denominator

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, b \neq 0$$

- *Example:* $\sqrt{\frac{100}{4}} = \frac{\sqrt{100}}{\sqrt{4}} = \frac{10}{2} = 5$ *Example:* $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$ Note that $\sqrt{5}$ cannot be simplified
- ***Simplified radicals do not contain radicals in the denominator.*** In order to simplify, use a process called ***rationalizing the denominator,*** whereby the numerator and denominator are both multiplied by the radical denominator in order to eliminate it. *Remember that a square root multiplied by itself is equal to the radicand.*

  Example: $\frac{2}{\sqrt{5}}$ To simplify, rationalize the denominator: $\frac{2}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

  Example: $\frac{3}{4+\sqrt{11}}$ To rationalize a denominator with two terms, multiply by the ***conjugate*** by changing the sign of the second term. Conjugates multiply as the difference of 2 squares, with the middle term dropping out.

$$\frac{3}{4+\sqrt{11}} \bullet \frac{4-\sqrt{11}}{4-\sqrt{11}} = \frac{3(4-\sqrt{11})}{16-4\sqrt{11}+4\sqrt{11}-11} = \frac{3(4-\sqrt{11})}{16-11} = \frac{3(4-\sqrt{11})}{5}$$

Rules for working with square roots when adding, subtracting, multiplying and dividing

- **Simplify the square roots.**
- **Perform the indicated operations.**
- **Simplify the final answer.**

Adding & Subtracting: Like square roots can be combined.

- $\sqrt{4} + \sqrt{9} = 2 + 3 = 5$
- $\sqrt{12} + \sqrt{3} = \sqrt{4 \bullet 3} + \sqrt{3} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$
- $2\sqrt{5} - \sqrt{5} = \sqrt{5}$
- $\sqrt{20} - \sqrt{5} = \sqrt{4 \bullet 5} - \sqrt{5} = 2\sqrt{5} - \sqrt{5} = \sqrt{5}$

Multiplying & Dividing: Use the product and quotient rules as outlined earlier. Unlike square roots ***can*** be multiplied and divided. Simplify first, then perform the multiplication or division, simplify the answer.

- *Examples*: $\sqrt{16}\sqrt{36} = 4 \bullet 6 = 24$ **,** $\sqrt{6}\sqrt{24} = \sqrt{144} = 12$ **,** $\sqrt{3}\sqrt{5} = \sqrt{15}$
- *Examples*: $\sqrt{\frac{72}{6}} = \sqrt{12} = \sqrt{4 \bullet 3} = 2\sqrt{3}$, $\frac{\sqrt{48}}{\sqrt{3}} = \frac{\sqrt{16 \bullet 3}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}} = 4$

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