



# Determinants

## Method #1

$$\begin{vmatrix} 4 & -1 & 2 \\ 6 & -1 & 0 \\ 1 & -3 & 4 \end{vmatrix} \leftarrow \text{Expand about the first row.}$$

Array of signs for top row

$$\begin{array}{c} + \quad - \quad + \\ \swarrow \quad \downarrow \quad \searrow \\ +(+4) \begin{vmatrix} -1 & 0 \\ -3 & 4 \end{vmatrix} \quad -(-1) \begin{vmatrix} 6 & 0 \\ 1 & 4 \end{vmatrix} \quad +(+2) \begin{vmatrix} 6 & -1 \\ 1 & -3 \end{vmatrix} \end{array}$$

$$D = \{4 [(4)(-1) - (0)(-3)]\} + \{(1) [(6)(4) - (0)(1)]\} + \{(2) [(6)(-3) - (-1)(1)]\}$$

$$D = 4(-4-0) + 1(24-0) + 2(-18-(-1))$$

$$D = -16+24+(-34)$$

$$\mathbf{D = -26}$$

## Method #2

Set the matrix up as shown below, repeating the first two columns.

$$\begin{vmatrix} 4 & -1 & 2 & 4 & -1 \\ 6 & -1 & 0 & 6 & -1 \\ 1 & -3 & 4 & 1 & -3 \end{vmatrix}$$

Draw diagonals as shown. Add the products of the straight-line diagonals and add the products of the dashed line diagonals, then find the difference.

$$D = \{(4)(-1)(4) + (1)(0)(1) + (2)(6)(-3)\} - \{(1)(-1)(2) + (-3)(0)(4) + (4)(6)(-1)\}$$

$$D = \{-16 + 0 + -36\} - \{-2 + 0 + -24\}$$

$$D = -52 - -26$$

$$\mathbf{D = -26}$$



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## Advanced Method #1

While it is customary to expand about the first row, it is possible to expand about any row or column. Whenever possible, choose a row or column that contains one or more zeros.\*

$$\begin{vmatrix} 4 & -1 & 2 \\ 6 & -1 & 0 \\ 1 & -3 & 4 \end{vmatrix} \leftarrow \text{Expand about the second row}$$

$$\begin{array}{ccc|c} + & - & + & \\ - & + & - & \text{Array of signs} \\ + & - & + & \end{array}$$
  
$$(-)6 \begin{vmatrix} -1 & 2 \\ -3 & 4 \end{vmatrix} + (+)(-1) \begin{vmatrix} 4 & 2 \\ 1 & 4 \end{vmatrix} + 0$$

\*Zero elements in the matrix result in a zero term.

$$D = \{(-6) [(4)(-1) - (-3)(2)]\} + \{(-1) [(4)(4) - (1)(2)]\} + \{0\}$$

$$D = (-6)[(-4) - (-6)] + -1[(16) - (2)] + 0$$

$$D = -12 + -14 + 0$$

$$D = -26$$