Concepts in Set Theory and Probability Theorems

Concepts in Set Theory

Set → A collection of well defined elements.

1. **Description** – A set defined in words.
   Example: Set A is the set of Natural numbers ending in 10.
2. **Roster** – A set is defined with a list of elements surrounded by braces {}.
   Example: A = {1,2,3,4,5,6,7,8,9,10}
3. **Set Builder Notation** –
   Example: A = {x|x is a natural number less than 11}, which reads: "Set A is set of all elements x such that x is a natural number less than 11."

Element → An item in a set denoted by the symbol $\in$.
Example: If A = {1,2,3}, then $3 \in A$

Equal sets → are identical, containing exactly the same elements.
Example: If, A = {A,B,C,D}, and B = {D,C,B,A} , then A = B

Equivalent sets → have the same cardinal number of elements, denoted by the symbol **n( )**, but the elements do not need to be identical.
Example: If A = {1,2,3,4} and B = {April, May, June, July}, then n(A) = n(B). Sets A and B are equivalent.

Empty or Null Set → is a set that contains no elements and are denoted by the symbols **{ }** and $\varnothing$.

Subset → denoted by the symbol $\subseteq$ occurs when all the elements of one set are also the elements of another. A subset may be, but doesn't have to be equal to the original set.
Example: If A = {A,B,C,D} and B = {A,B,C,D,E,F,G}, then $A \subseteq B$.

Proper Subset → denoted by the symbol $\subset$ occurs when the subset contains at least one less element than the original set.
Example: If A = {A,B,C,D} and B = {A,B,D}, then $B \subset A$

Number of Subsets → is 2^n, where n is the number of elements in the set.
Example: A = {A, B, C, D}. Since set A has 4 elements, the formula for number of subsets is: $2^4 = 16$. *Therefore, there are 16 subsets of set A. They are:* $\varnothing$, *{A}, {B}, {C}, {D}, {A,B}, {A,C}, {A,D}, {B,C}, {B,D}, {C,D}, {A,B,C}, {A,B,D}, {A,C,D}, {B,C,D} and {A,B,C,D}. Note that the first fifteen subsets of set A are also proper subsets. The formula for the number of proper subsets is* $2^n - 1$. *In this example of set A, the number of proper subsets is* $2^4 - 1 = 15$.

Concepts in Set Theory and Probability Theorems

Universal Set → contains all the elements for any specific discussion, and is symbolized by the symbol **U**.
Example: ***U*** *= {A,E,I,O,U}*

Intersection → contains the elements *common* to 2 or more sets and is denoted by the symbol, $\cap$.

Union → contains *all* the elements in two or more sets and is denoted by the symbol, $\cup$.

Complement → contains all the elements in the universal set that *are not* in the original set and is denoted by the symbol, ′.
Example: *U = {1,2,3,4,5,6,7,8,9,0}* *A= {1,2,3,}* *B = {2,3,4,5,6}*
A ∩ B = {2,3,} *A ∪ B = {1,2,3,4,5,6}* *A′ = {4,5,6,7,8,9,0}* *B′={1,7,8,9,0}*

Probability Theorems

Complement → $P(E) = 1 - P(E')$
$P(E') = 1 - P(E)$

Multiplication Rule → $P(A \cap B) = P(A) \times P(B)$
{AND, BUT, ALSO, AS WELL AS}

General Addition Rule → $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
{OR} Independent events, both events can occur

Special Addition Rule → $P(A \cup B) = P(A) + P(B)$
{OR} Events are mutually exclusive, and cannot occur simultaneously

Neither → $P(A \cup B)' = 1 - P(A \cup B)$
{NOT EITHER} Complement of the addition rule

Neither → $P(A \cup B)' = P(A') \cap P(B')$
{NOT EITHER} Only if independent events

Conditional → $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

{GIVEN THAT}

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